

PHYS 3717: Particle Physics

<https://homiller-hep.net/teaching/2026-2/>

Homework 5

Deadline: October 22, 2026 (in class)

Problem 1: Correlation Functions in ϕ^3 Theory

Consider ϕ^3 theory, defined by the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2 \phi^2 - \frac{1}{3!}\kappa \phi^3. \quad (1)$$

Compute the three-point function,

$$\langle \Omega | T \{ \phi(x) \phi(y) \phi(z) \} | \Omega \rangle, \quad (2)$$

in perturbation theory to first order in the coupling constant κ . Express your answer in terms of the coupling κ , Feynman propagators D_F , and integrals over the four-volume of spacetime. Represent each distinct term as a Feynman diagram.

Problem 2: Transfer Matrix Elements in ϕ^4 Theory

Consider the ϕ^4 theory studied in class, with the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2 \phi^2 - \frac{1}{4!}\lambda \phi^4. \quad (3)$$

To first order in the coupling constant λ , calculate the T matrix element for $2 \rightarrow 2$ scattering, $\phi(p_A) + \phi(p_B) \rightarrow \phi(p_C) + \phi(p_D)$, given by

$$\langle \vec{p}_C \vec{p}_D | iT | \vec{p}_A \vec{p}_B \rangle = \langle \vec{p}_C \vec{p}_D | \left[-i \int d^4x \frac{\lambda}{4!} N \{ \phi_I^4(x) \} \right] | \vec{p}_A \vec{p}_B \rangle, \quad (4)$$

where $N\{\dots\}$ denotes the normal ordering operator. Evaluate this expression by writing out the normal ordered products of ϕ_I^+ and ϕ_I^- and expanding them as superpositions of creation/annihilation operators. Use the commutation relations of the creation / annihilation operators to evaluate the resulting matrix element, and then carry out the integrals over three momenta and the four-volume of spacetime. It may be useful to employ the abbreviated notation below when working through the algebra,

$$[a_{\vec{p}_i}, a_{\vec{p}_j}^\dagger] = (2\pi)^3 \delta(\vec{p}_i - \vec{p}_j) \equiv \hat{\delta}_{ij} \quad (5)$$

Problem 3: Cosmic Ray Muon Lifetime

Relativistic muons and pions are produced in cosmic ray collisions in the upper atmosphere, with a typical height of about $h = 15$ km above sea level.

- (a) Assuming the particles travel directly downwards from the point of their production, show that the fraction of particles that survive to sea level is given by

$$\frac{N(0)}{N(h)} = \exp \left[-\frac{h}{\gamma\beta c\tau} \right], \quad (6)$$

where τ is the proper lifetime of the particle, $\beta = v/c$ is the velocity, and $\gamma = (1-\beta^2)^{-1/2}$ is the Lorentz boost factor.

- (b) Estimate the fraction of muons with mean energy $E = 1$ GeV that survive to sea level.
- (c) Estimate the fraction of muons with mean energy $E = 10$ GeV that survive to sea level.
- (d) Estimate the fraction of charged pions $\pi^\pm$ with mean energy $E = 10$ GeV that survive to sea level.
- (e) Estimate the fraction of neutral pions π^0 with mean energy $E = 10$ GeV that survive to sea level.