

PHYS 3102: Effective Field Theories in Particle Physics

Samuel Homiller

<https://homiller-hep.net/teaching/2026-1/>

Homework 3

Deadline: 03/20/2026

Problem 1: QED Corrections to Muon Decay

In class we considered QCD corrections to the decay of a free b quark, $b \rightarrow c\bar{u}d$, and found that the relevant four-fermion operators receive divergent contributions that must be removed via operator renormalization.

Now, compute the UV-divergent part of the one-loop QED corrections to the decay $\mu \rightarrow e\nu_\mu\bar{\nu}_e$. Show that the UV divergences cancel, so that this four-fermion operator is not renormalized. Explain why.

Problem 2: Renormalization of a Toy Yukawa Theory

Consider the toy Yukawa theory we discussed in class, with the fermion taken to be massless ($m \rightarrow 0$):

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi + \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}M^2\phi^2 - g\phi\bar{\psi}\psi. \quad (1)$$

1. Compute the divergent part (including the $\log\mu^2$ piece) of the two-point correlation functions of the two fields at one-loop order. Use this to determine the anomalous dimensions of each field.
2. Compute the divergent part of the one-loop scalar–fermion vertex function. Derive the beta function for g from your result.
3. Determine the running of the Yukawa coupling g . Solve for the value of μ where the one-loop value of $g(\mu)$ diverges, the “Landau pole” of the coupling, and rewrite $g(\mu)$ in terms of this mass scale.

Problem 3: Parity and CP of the Meson Fields

In this problem, we’ll work out the transformations of the matrix field Σ from chiral perturbation theory under parity P and charge conjugation, C .

Starting with the operations of P and C on fermions:

$$\begin{aligned} P : \psi &\mapsto \gamma^0\psi, & C : \psi &\mapsto C\bar{\psi}^T, \\ \text{with } C^\dagger = C^{-1} = C^T = -C, & & C^{-1}\gamma_\mu C &= -\gamma_\mu^T, & C^{-1}\gamma_5 C &= \gamma_5, \end{aligned} \quad (2)$$

where X^T indicates the transpose of X , find how Σ transforms under P and C . What does this imply for the ϕ^a mesons? Which of the mesons are eigenstates of CP , and are they CP even, or odd?