PHYS 3102: Effective Field Theories in Particle Physics

Samuel Homiller
https://homiller-hep.net/teaching/2026-1/

Homework 2

Deadline: 02/27/2026

Problem 1: Proton Decay from Baryon-Number Violating Operators
Consider the B-violating operator

$$\mathcal{O}_{duQ} = \epsilon^{\alpha\beta\gamma}\epsilon_{ab}(\bar{d}^\dagger_\alpha \bar{u}^\dagger_\beta)(Q^a_\gamma L^b) \tag{1}$$

with α, β, γ color fundamental indices and a, b $\mathrm{SU}(2)_L$ fundamental indices.

1. Considering only the first generation, write this operator out in terms of its $\mathrm{SU}(2)_L$ component fields, and find a resulting 2-body decay process for the proton.
2. Using only dimensional analysis, estimate the lifetime for this decay as a function of the scale Λ.[1]
3. Find the current limit on this decay mode from the PDG,[2] and compare this to your estimate above to get a lower limit on the scale Λ.

Problem 2: Minimal Flavor Violation and Kaon-Mixing
In this problem, we'll work out more detail how the MFV Ansatz alleviates strong constraints on new physics from flavor-violating operators. Consider the operator,

$$\mathcal{O}^{(1)}_{QQ,1212} = (Q^\dagger_1 \bar{\sigma}^\mu Q_2)(Q^\dagger_1 \bar{\sigma}_\mu Q_2)\,. \tag{2}$$

Assuming the Wilson coefficient is ~ 1, a global fit to the SM flavor sector including Kaon-mixing data (see e.g., [arXiv:0707.0636]) sets a naive constraint $\Lambda \gtrsim 1.5 \times 10^4$ TeV.

1. Work out the appropriate transformation for this operator under the SM flavor group and identify the tensor products of the SM Yukawa matrices that have the same transformation rules.
2. In the limit where only the top quark mass is nonzero (but all the CKM entries are non-vanishing), find the expected scaling of the Wilson coefficient in the MFV hypothesis.
3. Using the MFV value of the Wilson coefficient, work out a new bound on the scale Λ.

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[1] A realistic calculation involves not only the proton mass, but also the pion decay constant and a hadronic matrix element that must be computed on the lattice. Here, I'm only looking for a rough estimate using the most relevant scale in the decay.

[2] Hint: the most stringent limit was set in 2020.

Problem 3: Tree-Level Matching Examples

Here we will consider two simple extensions of the Standard Model, and perform the tree-level matching onto the dimension-six operators of the Standard Model EFT. In each case, you should:

1. Find the equations of motion for the new, heavy fields, and use these to integrate out the heavy fields and obtain an effective action.
2. Perform any necessary manipulations to write the effective action in terms of Warsaw basis operators. Express the Wilson coefficients of the Warsaw basis operators in terms of the UV model parameters.
3. Finally, write down the Feynman diagrams that would have led to the same result in a diagrammatic matching calculation.

Example 1: A Singlet Scalar:

Consider first a heavy, real singlet scalar ϕ, with mass M that couples to the SM with the interactions:

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}M^2\phi^2 - A\phi(H^\dagger H) - \frac{1}{2}\kappa\phi^2(H^\dagger H) - \frac{1}{3!}\mu_\phi\phi^3 - \frac{1}{4!}\lambda_\phi\phi^4 \tag{3}$$

Note that A, μ_ϕ are dimensionful parameters, which allows for formally dimension-6 contributions that do not scale as $1/M^2$.

Example 2: A Vector-Like Top Partner:

Now consider a vector-like, $\mathrm{SU}(2)_L$ singlet top partner. This can be written in terms of a pair of Weyl fermions T, $\bar{T}$ where $\bar{T}$ has the same quantum numbers as $\bar{u}$ in the Standard Model. For simplicity, neglect quark mixing and consider only interactions with the third generation of the Standard Model. The relevant interactions are[3]

$$\mathcal{L} \supset MT\bar{T} + y_t HQ_3\bar{u}_3 + \lambda_T HQ_3\bar{T}\,. \tag{4}$$

Assume $M \gg v$. The parameters y_t, λ_T are dimensionless.

[3] An additional mass term $\sim T\bar{u}$ is allowed by the gauge symmetries but can be removed by a redefinition of the fields $\bar{u}$ and $\bar{T}$.